

Pseudospin–Electron Model in Large Dimensions

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Abstract. Energy spectrum and thermodynamics of the pseudospin–electron model introduced at the consideration of the anharmonicity effects in high- T_c superconductors are investigated in the dynamical mean field approximation ($d = \infty$ limit). In the limit of zero electron correlation $U \rightarrow 0$ this model is analytically exactly soluble within this approach: in the $\mu = \text{const}$ regime the first order phase transition with the jump of the pseudospin mean value $\langle S^z \rangle$ and reconstruction of the electron spectrum can realize, while in the $n = \text{const}$ regime the phase separation in electron subsystem can take place for certain values of the model parameters. On the basis of the obtained results the applicability of the approximate schemes previously used for the investigation of the pseudospin–electron model are discussed.

PACS. 71.10.Fd Lattice fermion models (Hubbard model, etc.) – 71.38.+i Polarons and electron–phonon interactions – 77.80.Bh Phase transitions and Curie point – 63.20.Ry Anharmonic lattice modes

1 Introduction

The theoretical investigation of the strongly correlated electron systems is an enduring subject of interest in condensed matter physics especially during last ten years after the discovery of high- T_c superconductivity. Recent studies of strongly correlated Hubbard type models elucidate some important features of high- T_c , namely the d -wave pairing and the role of antiferromagnetic fluctuations [1]. However some features of the cuprates are not well understood (e.g. the existence of an “optimal” doping, the effect of charge fluctuations, strong electron–phonon interaction, lattice dynamics and instabilities of ferroelectric type). Thus the extensions of the Hubbard model by the van Hove scenario concept, the incorporation of the local electron–phonon interaction (the Hubbard–Holstein model for harmonic phonons and the pseudospin–electron model [2] for anharmonic ones), generalization to a two or three band model, the inclusion of intersite electron interaction, etc. are under consideration.

Within these models the pseudospin–electron one [2] in a simplest way includes the interaction of correlated electrons with some local lattice excitations described by pseudospins (e.g. anharmonic vibrations of apex oxygen in YBaCuO type HTSC’s), and shows the possibility of dipole (pseudospin) and charge density instabilities [3,4] and phase separation [5] due to the effective retarded interaction between pseudospins via conducting electrons. All these results were obtained within the generalized random field approximation (GRPA) [6] which is a realization of the appropriate perturbation theory for correlation functions in the case of strong coupling ($U \gg t$) and corresponds to the mean field approximation in calculation of mean values. There are no good criteria of its applicability

and it is supposed that GRPA gives correct description in the case of large dimensionality of local (site) states.

In recent years the essential achievements of the theory of strong correlated electron systems are connected with the development of the dynamical mean field theory (DMFT) proposed by Metzner and Vollhardt [7] for Hubbard model (see also [8] and references therein). DMFT is a nonperturbative scheme which allows to project Hubbard model on the single impurity Anderson model and is exact in the limit of infinite space dimensions ($d = \infty$). Moreover, some class of models (e.g. Falicov–Kimball model [9]) can be studied almost analytically within DMFT.

Here we apply DMFT to the investigation of pseudospin–electron model in the limit of zero electron correlation ($U = 0$) which can be treated analytically.

2 Perturbation theory in terms of electron transfer

The Hamiltonian of pseudospin–electron model in the absence of electron correlations can be written in the form:

$$H = \sum_i H_i + \sum_{ij\sigma} t_{ij} a_{i\sigma}^\dagger a_{j\sigma}, \quad (1)$$

where

$$H_i = g S_i^z \sum_{\sigma} n_{i\sigma} - \mu \sum_{\sigma} n_{i\sigma} - h S_i^z \quad (2)$$

is single-site Hamiltonian, and includes local interaction of conducting electrons with pseudospins placed in longitudinal field h (asymmetry parameter of anharmonic potential).

In general, one–electron Green’s function $G_\sigma(\omega_n, \mathbf{k})$

$$G_{ij}^\sigma(\tau - \tau') = \left\langle \mathcal{T} a_{i\sigma}(\tau) a_{j\sigma}^\dagger(\tau') \sigma(\beta) \right\rangle_0 / \langle \sigma(\beta) \rangle_0 \quad (3)$$

$$\sigma(\beta) = \mathcal{T} \exp \left\{ - \int_0^\beta d\tau \sum_{ij\sigma} t_{ij} a_{i\sigma}^\dagger(\tau) a_{j\sigma}(\tau) \right\} \quad (4)$$

satisfies Larkin’s equation

$$G_{ij}^\sigma(\tau - \tau') = \Xi_{ij}^\sigma(\tau - \tau') + \Xi_{il}^\sigma(\tau - \tau'') t_{lm} G_{mj}^\sigma(\tau'' - \tau'), \quad (5)$$

where summation (integration) over repeated indices is supposed. The formal solution of eq. (5) can be written in the form

$$G_\sigma(\omega_n, \mathbf{k}) = \frac{1}{\Xi_\sigma^{-1}(\omega_n, \mathbf{k}) - t_\mathbf{k}} \quad (6)$$

and the task is to calculate the irreducible according to Larkin parts $\Xi_\sigma(\omega_n, \mathbf{k})$.

It is convenient to introduce projective operators on pseudospin states

$$P_i^\pm = \frac{1}{2} \pm S_i^z, \quad (P_i^\pm)^2 = P_i^\pm, \quad P_i^+ P_i^- = 0 \quad (7)$$

and by substitution $P_i^+ = c_i$, $P_i^- = 1 - c_i$ Hamiltonian (1), (2) can be transformed into the Hamiltonian of binary alloy. On the other hand, if we keep in (1), (2) only electrons with one orientation of spin by removing the sum over spin indices and putting $\sigma = \uparrow$ and consider electrons with $\sigma = \downarrow$ as localised $P_i^+ = n_{i\downarrow}$, $P_i^- = 1 - n_{i\downarrow}$ we get the Hamiltonian of the Falicov–Kimball model where h plays a role of the chemical potential for the localized \downarrow -electrons. As a rule the common chemical potential is introduced for both electron subsystems but the case of two chemical potentials was also considered (see, e.g. [10]) and the first consideration of Falicov–Kimball model within DMFT was done by Brandt and Mielsch [9].

Formally diagrammatic series for the irreducible part $\Xi_{ij}^\sigma(\omega_n)$ are the same for all these models

$$\Xi_{ij}^\sigma(\omega_n) = \left(\text{diagram 1} + \text{diagram 2} + \dots \right) \delta_{ij} + \text{diagram 3} + \dots \quad (8)$$

and includes both single–site and intersite contributions. Here, arrows indicate electron propagators $g_{\sigma i}^\pm(\omega_n) = \frac{P_i^\pm}{i\omega_n + \mu \mp \frac{h}{2}}$ in the subspaces projected on the pseudospin states of site i and ovals represent semi–invariant averaging of projection operators.

The main difference between these models is in the way how an averaging procedure over projection operators is performed (thermal statistical averaging in the case of pseudospin–electron and Falicov–Kimball models and configurational averaging for binary alloy) and how self–consistency is achieved (fixed value of longitudinal field h

for pseudospin–electron model, fixed value of the component concentration c for binary alloy and fixed value of the electron concentration — total or for both electron subsystems — for Falicov–Kimball model).

3 The limit of large dimensions ($d \rightarrow \infty$)

In the case of high dimensions ($d \rightarrow \infty$) one should scale hopping integral

$$t_{ij} \rightarrow \frac{t_{ij}}{\sqrt{d}} \quad (9)$$

in order to obtain finite density of states (the Gaussian one for $d = \infty$ hypercubic lattice $\rho(\varepsilon) = \frac{1}{W\sqrt{\pi}} e^{-\varepsilon^2/W^2}$ and semi–elliptic d.o.s. for $d = \infty$ Bethe lattice $\rho(\varepsilon) = \frac{2}{\pi W^2} \sqrt{W^2 - \varepsilon^2}$ [8]). Due to such scaling only single–site contributions survive in the expression for irreducible parts Ξ_σ

$$\Xi_{ij}^\sigma(\tau - \tau') = \delta_{ij} \Xi_\sigma(\tau - \tau'), \quad \Xi_\sigma(\omega_n, \mathbf{k}) = \Xi_\sigma(\omega_n) \quad (10)$$

and such site–diagonal function, as it was shown by Brandt and Mielsch [9], can be calculated by mapping the infinite–dimensional lattice problem on the atomic model

$$e^{-\beta H} \rightarrow e^{-\beta H_{\text{eff}}} = e^{-\beta H_0} \quad (11)$$

$$\times \mathcal{T} \exp \left\{ - \int_0^\beta d\tau \int_0^\beta d\tau' \sum_\sigma J_\sigma(\tau - \tau') a_\sigma^\dagger(\tau) a_\sigma(\tau') \right\}$$

with auxiliary Kadanoff–Baym field $J_\sigma(\tau - \tau')$ [11] which has to be self consistently determined from the condition that the same function Ξ_σ defines Green’s functions for lattice (6) and atomic limit

$$G_\sigma^{(a)}(\omega_n) = \frac{1}{\Xi_\sigma^{-1}(\omega_n) - J_\sigma(\omega_n)}. \quad (12)$$

“Dynamical” mean field $J_\sigma(\tau - \tau')$ describes the hopping (transfer) of electron from atom into environment at moment τ , propagation in environment without stray into atom until moment τ' . Connection between these “dynamical” mean field of atomic problem and Green’s function of the lattice can be obtained using standard CPA approach [8]:

$$J_\sigma(\omega_n) = \Xi_\sigma^{-1}(\omega_n) - G_\sigma^{-1}(\omega_n), \quad (13)$$

where

$$G_\sigma^{(a)}(\omega_n) = G_\sigma(\omega_n) = \int_{-\infty}^{+\infty} dt \frac{\rho(t)}{\Xi_\sigma^{-1}(\omega_n) - t} \quad (14)$$

is a single–site Green’s function both for atomic limit and lattice. Here summation over wave vector was changed by the integration with the density of states $\rho(t)$.

In order to complete our self–consistent set of equations we should find expression for Green’s function in the atomic limit (12). Due to the properties of the projection operators (7) one can rewrite Hamiltonian of atomic problem (11) in the form

$$e^{-\beta H_{\text{eff}}} = P^+ e^{-\beta H^{(+)}} + P^- e^{-\beta H^{(-)}} \quad (15)$$

and our space of states splits into two independent subspaces hence all projection operators (7) act at the same site and in any order of the perturbation theory expansion all projection operators can be replaced by their product result and there are no necessity to make semi-invariant expansions.

Single-electron Green's function is a sum of Green's functions in subspaces and is equal

$$G_{\sigma}^{(a)}(\omega_n) = \frac{\langle P^+ \rangle}{i\omega_n + \mu - J_{\sigma}(\omega_n) - \frac{g}{2}} + \frac{\langle P^- \rangle}{i\omega_n + \mu - J_{\sigma}(\omega_n) + \frac{g}{2}}. \quad (16)$$

Partition functions in subspaces are

$$Z_{\pm} = \text{Sp} e^{-\beta H_{\pm}} = e^{\pm \frac{\beta h}{2} - Q_{\pm}} \quad (17)$$

$$= e^{\pm \frac{\beta h}{2}} \prod_{\sigma} \left(1 + e^{-\beta(\mu \mp \frac{g}{2})} \right) \prod_n \left(1 - \frac{J_{\sigma}(\omega_n)}{i\omega_n + \mu \mp \frac{g}{2}} \right)$$

and presents the partition functions of the non-interacting fermions with frequency dependent hopping placed in the external field formed by pseudospin.

Pseudospin mean value is determined by equation

$$\langle S^z \rangle = \frac{1}{2} \frac{Z_+ - Z_-}{Z_+ + Z_-} \quad (18)$$

$$= \frac{1}{2} \tanh \frac{1}{2} (\beta h - (Q_+[\langle S^z \rangle] - Q_-[\langle S^z \rangle]))$$

which is an analogue of the well known equation of state for Ising model in mean-field approximation: $\langle S^z \rangle = \frac{1}{2} \tanh \frac{\beta}{2} (h + J_0 \langle S^z \rangle)$. It should be noted that in the case of Lorentzian density of states $\rho(\varepsilon) = \frac{W}{\pi(W^2 + \varepsilon^2)}$, which is frequently used in some applications of DMFT, one can easily obtain a simple result $J_{\sigma}(\omega_n) = iW$ [8], quantities Q^{\pm} do not depend on $\langle S^z \rangle$ and equation (19) transforms into an expression for $\langle S^z \rangle$ that indicates the sensitivity of the equation of state to the shape of d.o.s.

Electron concentration mean value is determined by

$$\langle n \rangle = \frac{1}{\beta} \sum_{m\sigma} G_{\sigma}(\omega_m) \quad (19)$$

and the functional of thermodynamic potential can be derived in the same way as it was done in [9] for Falicov–Kimball model

$$\frac{\Omega}{N} = \Omega_{(a)} - \frac{1}{\beta} \sum_{n\sigma} \left\{ \ln G_{\sigma}^{(a)}(\omega_n) - \frac{1}{N} \sum_{\mathbf{k}} \ln G_{\sigma}(\omega_n, \mathbf{k}) \right\}, \quad (20)$$

where

$$\Omega_{(a)} = -\frac{1}{\beta} \ln(Z_+ + Z_-) \quad (21)$$

is a thermodynamic potential for atomic problem.

Below, all calculations will be performed for semi-elliptic density of states when the auxiliary field is determined by the simple cubic equation

$$J_{\sigma}(\omega_n) = \frac{W^2}{4} \left\{ \frac{\langle P^+ \rangle}{i\omega_n + \mu - J_{\sigma}(\omega_n) - \frac{g}{2}} \right. \quad (22)$$

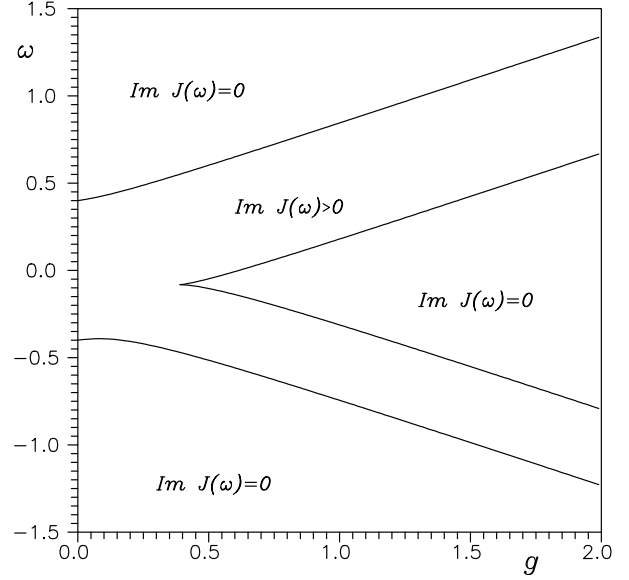


Fig. 1. Electron bands boundaries (semi-elliptic d.o.s., $W = 0.4$, $\langle S^z \rangle = 0.2$).

$$+ \frac{\langle P^- \rangle}{i\omega_n + \mu - J_{\sigma}(\omega_n) + \frac{g}{2}} \Big\}.$$

In a usual way we perform analytical continuation on real axis ($i\omega_n \rightarrow \omega - i\delta$) and only solutions of (22) with $\Im m J_{\sigma}(\omega) > 0$ must be considered. Band boundaries are determined from the condition $\Im m J_{\sigma}(\omega) \rightarrow 0$ and in Fig. 1 their dependence on coupling constant g are presented. One can see that there exists critical value of coupling constant $g \sim W$ when a gap in spectrum appears. It should be noted that within GRPA as well as in other approaches where single-electron Green's function is calculated in Hubbard–I approximation, when we keep only the first term of the single-site contribution in the expression for the irreducible part (8), these gap in spectrum always exists.

In the case of strong coupling ($g \gg W$) an analytical solutions can be obtained

$$J_{\sigma}(\omega) = \frac{1}{2} \left(\omega \mp \frac{g}{2} \right) + \frac{i}{2} \sqrt{W^2 \langle P^{\pm} \rangle - \left(\omega \mp \frac{g}{2} \right)^2} \quad (23)$$

for upper and lower subbands, respectively, and one can see that subbands halfwidth is equal to $W\sqrt{\frac{1}{2} \pm \langle S^z \rangle}$ whereas in Hubbard–I approximation it is $W(\frac{1}{2} \pm \langle S^z \rangle)$. This result clearly shows that even for the case of strong coupling when subbands are well separated and one of them become narrow ($\langle S^z \rangle \rightarrow \pm \frac{1}{2}$) Hubbard–I approximation is insufficient and can not be derived from the exact solution in any way, e.g. due to the subbands halfwidth square root dependence on the localized states occupancy ($\langle P^{\mp} \rangle \rightarrow 0$).

Presented above expressions were obtained for the fixed value of the chemical potential μ when stable states are determined from the minimum of the thermodynamical potential (20). This regime $\mu = \text{const}$ corresponds to

the case when the charge redistribution between conducting sheets CuO_2 and other structural elements (charge reservoir, e.g. nonstoichiometric in oxygen CuO chains in YBaCuO type structures) which fix the value of the chemical potential is allowed. On the other hand, in the regime of the fixed electron concentration value one should solve equation for chemical potential $n = \langle n \rangle$ (19) and stable states are determined by the minimum of the free energy $F = \Omega + \mu n$.

4 Results and discussion

Integrals in Eqs. (17) and (20) can be calculated analytically for states with $\langle S^z \rangle = \pm \frac{1}{2}$ at zero temperature and corresponding phase diagrams $\mu - h$ which indicate stability regions for these states are shown in Fig. 2a and b for $g > W$ and $g < W$, respectively. One can see two regions of μ and h values where the states with $\langle S^z \rangle = \pm \frac{1}{2}$ coexists. In the vicinity of these regions the phase transitions of first order with the change of the longitudinal field h and/or chemical potential μ take place (see Fig. 3) and they are shown by thick lines on phase diagrams (Fig. 2).

There are no any specific behaviour when chemical potential is placed out of bands. If chemical potential is placed in upper subband the graphs presented in Fig. 3 transform according to the internal symmetry of the Hamiltonian:

$$\mu \rightarrow -\mu, \quad h \rightarrow 2g - h, \quad n \rightarrow 2 - n, \quad S^z \rightarrow -S^z. \quad (24)$$

With the temperature increase the region of the phase coexistence narrows and the corresponding phase diagram $T_c - h$ is shown in Fig. 4. One can see that with respect to Ising model the phase coexistence curve is shifted in field and distorted from the vertical line and hence the possibility of the first order phase transition with the temperature change exists in pseudospin–electron model for the narrow range of h values.

As it was mentioned above, the band structure is determined by the pseudospin mean value and its change is accompanied by the corresponding changes of the electron concentration and for the (μ, h) values fixed on the first order phase transition line there are three solutions for electron concentration one of which is unstable.

In the case of the fixed value of the electron concentration value (regime $n = \text{const}$) this first order phase transition transforms into the phase separation. One can see regions with $d\mu/dn \leq 0$, which correspond to this effect in electron subsystem, on the concentration dependencies (Figs. 5 and 6a).

The corresponding dependencies of free energy $F = \Omega + \mu n$ are given in Fig. 6b. In the phase separated region free energy deflects up and concentration values at binodal points are determined by the tangent line touch points or from the chemical potential dependencies (Fig. 6a) using Maxwell construction. Resulting phase diagram $T - n$ is shown in Fig. 7.

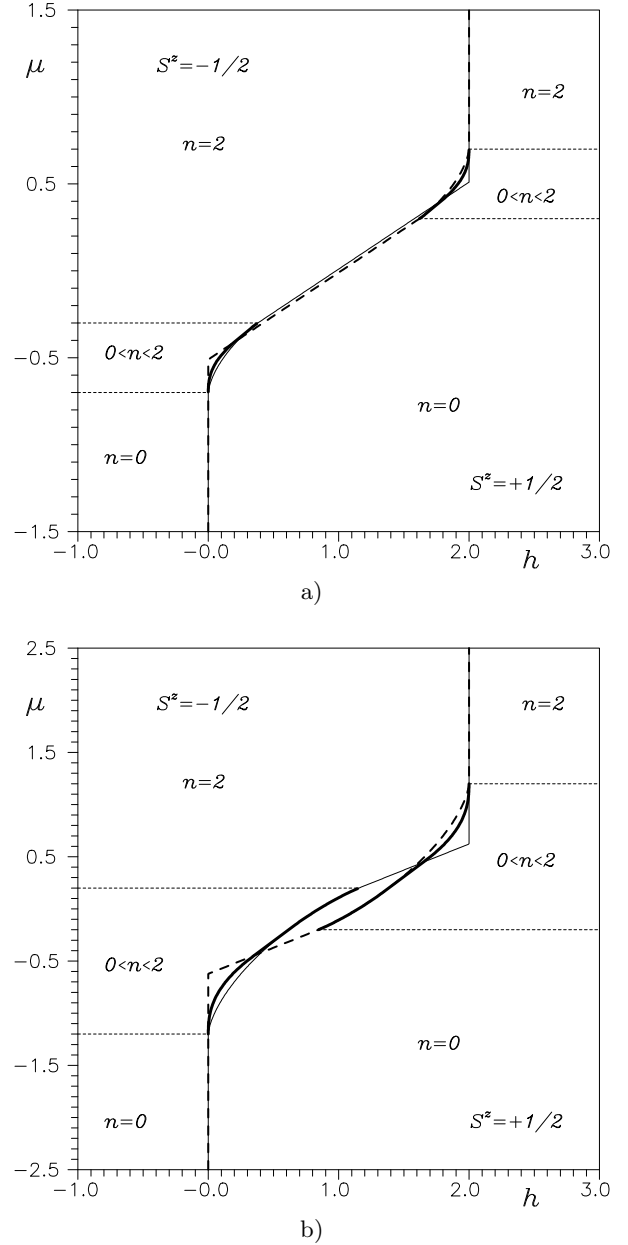


Fig. 2. Phase diagram $\mu - h$. Dashed and thin solid lines surround regions with $S^z = \pm \frac{1}{2}$, respectively. Thick solid line indicate the first order phase transition points. a) $g = 1$, $W = 0.2$; b) $g = 1$, $W = 0.7$.

For the first time the possibility of phase separation in pseudospin–electron model was marked in [5] where it was obtained within GRPA in the limit of strong correlation $U \rightarrow \infty$. Here it is observed for the opposite case of $U = 0$.

The problem of phase separation in strongly correlated systems is not new (see [1] and references therein). It was shown for Hubbard and $t - J$ models [12] that for some parameter values system separates into hole-rich and hole-poor regions with paramagnetic and antiferromagnetic orders, respectively, and long-range interaction between these charged regions is considered as an origin of the appearance of stripe structures. In Ref. [10] the phase

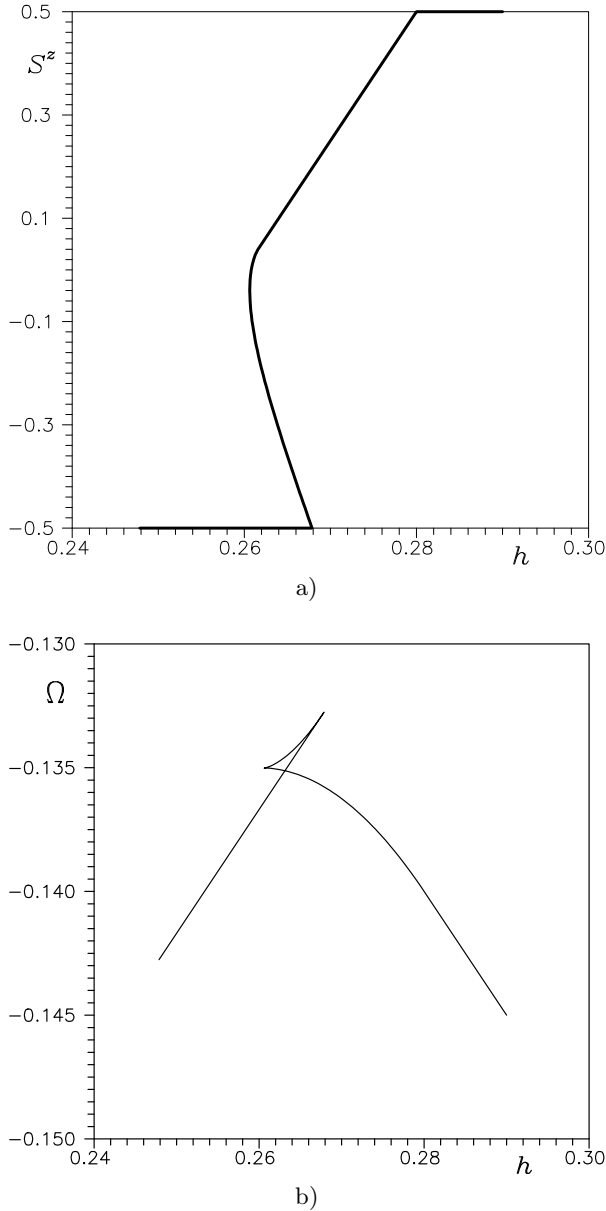


Fig. 3. Field dependencies of $\langle S_z \rangle$ (a) and thermodynamical potential (b) for $\mu = \text{const}$ regime when chemical potential is placed in the lower subband $\mu = -0.37$ ($W = 0.2$, $g = 1$, $T = 0$).

segregation for some parameter values was reported for the annealed binary alloy with diagonal disorder described by Falicov–Kimball model. In our case of pseudospin–electron model without electron correlations system separates into regions with different values of electron concentration and pseudospin mean value and electron spectrum contains both wide empty electron band and occupied localized states of the regions with $n \sim 0$ as well as partially filled wide electron band and empty localized states of the regions with $n \sim 1$ (see Fig. 5) the weights of which are determined by the electron concentration. Such type localized states (polarons) results from the strong electron–out of plane apical oxygen vibrations coupling ($g > W$) in the

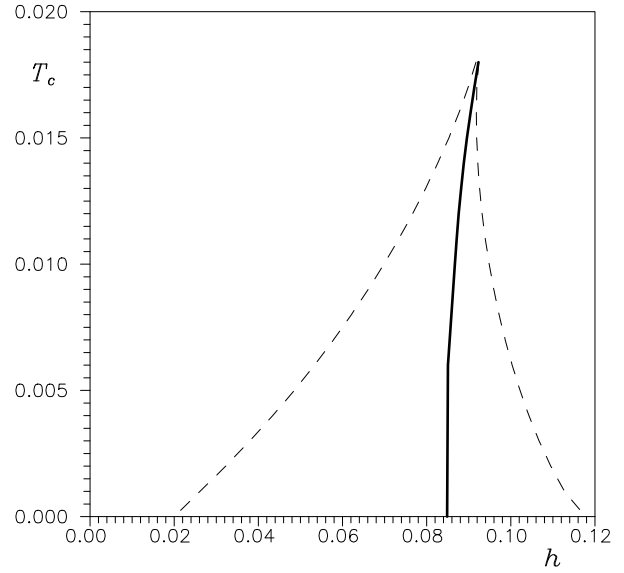


Fig. 4. Phase diagram $T_c - h$: solid and dashed lines indicate the first order phase transition line and boundaries of the phase stability, respectively ($g = 1$, $W = 0.2$, $\mu = -0.5$)

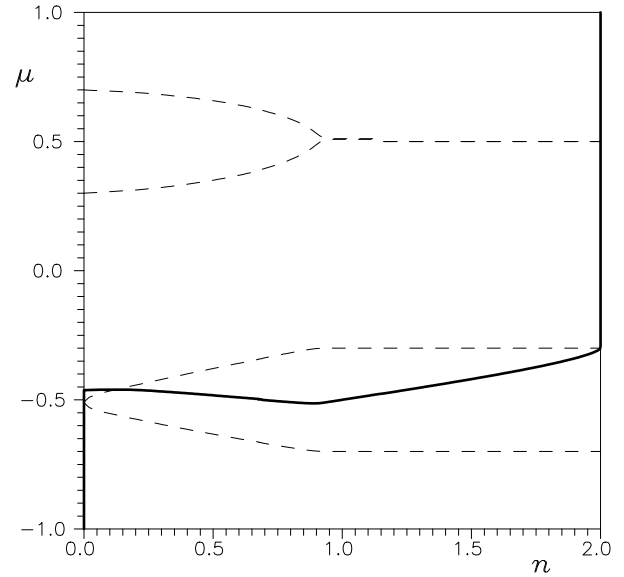


Fig. 5. Dependence of the chemical potential μ and electron bands boundaries (dashed lines) on the electron concentration n ($T = 0.001$, $g = 1$, $W = 0.2$, $h = 0.1$).

case of YBaCuO–type structures and it is supposed that the hopping between them gives significant contribution in the carrier relaxation observed by the resonant Raman spectroscopy [13].

It should be noted that in the case of spinless fermions Hamiltonian (1) can be applied for the description of the oxygen vacancies subsystem in high- T_c superconductors, which can be treated as quasiequilibrium, and it is known that their interaction with some relaxation type lattice mode leads to the phase separation and appearance of superstructures and stripes [14].

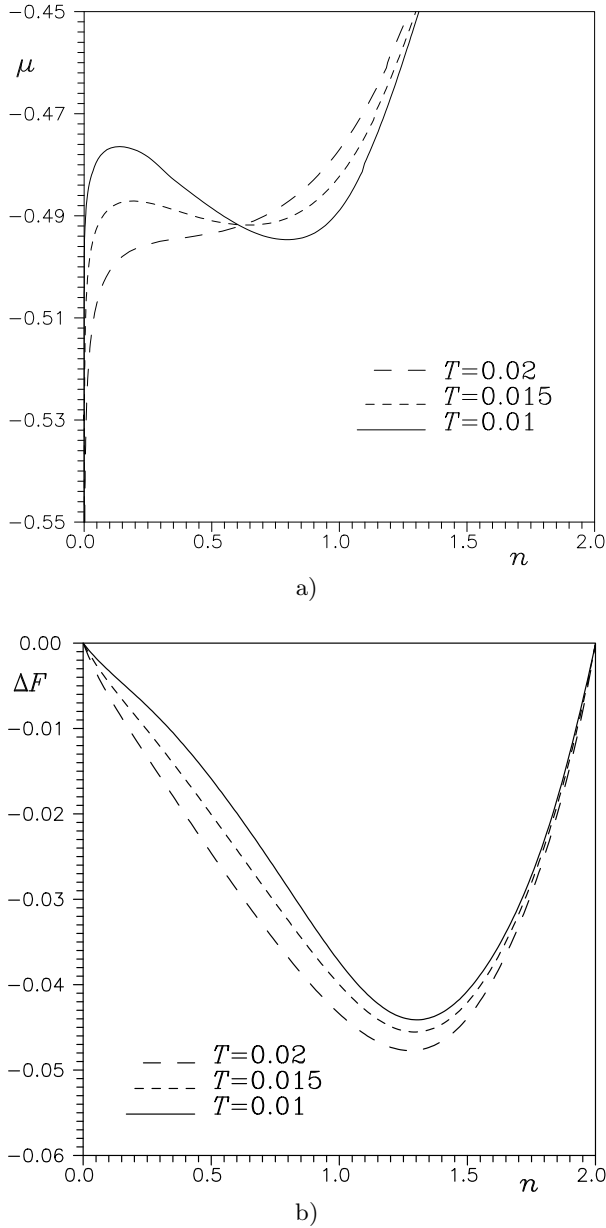


Fig. 6. Dependence of the chemical potential μ (a) and deviation of free energy from linear dependence $\Delta F = F(n) - \frac{n}{2}F(2) - (1 - \frac{n}{2})F(0)$ (b) on the electron concentration n for different temperatures T ($g = 1$, $W = 0.2$, $h = 0.1$).

In this paper we investigated the possible phase transitions in pseudospin–electron model within DMFT without creation of super structures ($\mathbf{k} = 0$) and the phase diagrams presented in Figs. 4 and 7 concern only this case. In order to detect instabilities associated with a specific wave vectors one should calculate response functions which will be the subject of the further investigations.

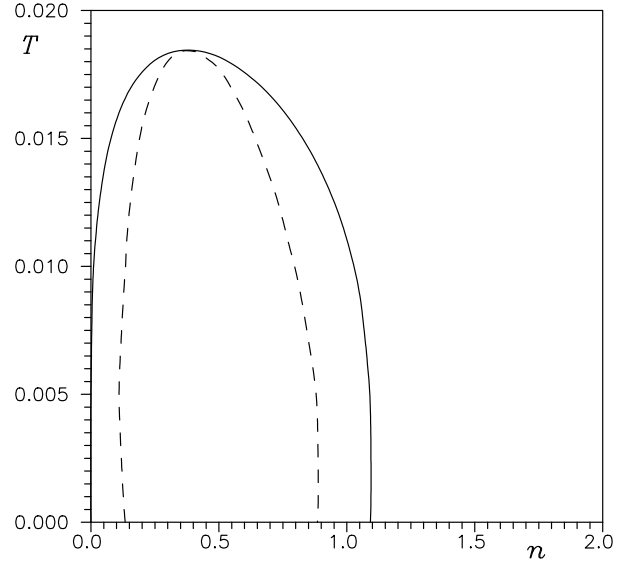


Fig. 7. Phase diagram $T - n$ for phase separated state: solid line — binodal, dashed line — spinodal ($g = 1$, $W = 0.2$, $h = 0.1$).

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